Issues Arising When Teachers Make Extensive Use of Computer Algebra in their Mathematics Lessons

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This paper focuses on two teachers, who had hitherto made no use of Derive with their classes, who agreed to make regular use of Derive with one of their classes over the course of one year. The teachers present an account of what they did. This forms the basis for a discussion of time issues in using Derive, the use of materials, pressures on teachers which influence their use of Derive and the 'immediacy' of Derive.

1. INTRODUCTION

Teachers are being encouraged, 'pushed' may be a better word, to make use of computers in their mathematics lessons. Schools in England are, as we write, signing up for training via the New Opportunities Fund (http://www.nof.org.uk/) and schemes such as the National Grid for Learning (http://www.rm.com/ngfl/). These are merely English versions of education policy moves that are afoot around the globe. Stephen and Steve's response was *OK, let's have a go, I'll try Derive.* A project at a local university acted as a catalyst for and a means of monitoring the work. This paper explores issues in teaching and aspects of student learning.

A few notes on the decision to use *Derive* are in order. Both Stephen and Steve initially focused on *Derive* but both ended up (details are provided below) using other ICT[1] tools as well as *Derive*. We are not sure that clear reasons for the decision to use *Derive*, rather than another computer algebra system, can be isolated. Factors which impinged on this choice, on reflection, were: personal familiarity; a belief that *Derive* was easier for High School students to learn than, say, Maple; a small site licence for *Derive* was cheaper than buying a class set of TI-92s or TI-89s. Although *Derive* is the CAS referred to in this paper, we believe that most of the comments made about it here apply similarly to other CAS.

Before going into details of what emerged we provide a brief literature review on mathematics teachers using technology, with a particular focus on computer algebra, and an overview of the project.

2. TEACHERS USING TECHNOLOGY

Considerable research on students' understanding of mathematics using ICT exists. Research on teachers' ICT practices in mathematics classes is, however, a relatively recent phenomenon. Bottino & Furinghetti (1996) focus on secondary mathematics teachers' roles when faced with ICT curriculum reform and conclude that "The introduction of informatics in mathematics teaching works only when it is perceived as an answer to questions already present in teachers' minds." Farrell (1996) focuses on the roles and behaviours of students and teachers in ICT mathematics lessons. An analysis of video-tapes produced evidence of a partial shift in the teachers' roles from manager to fellow investigator and a partial shift in students' roles from 'seatwork' to task setter. Moreira & Noss (1995) examined primary teachers' attitudes to change in an ICT environment following in-service training. They effectively argue teachers' attitudes to and beliefs about ICT-use cannot be abstracted from their situations, contexts and cultures, and that time is an important factor. Thompson (1992) supports this line of inquiry arguing that the quality of mathematics taught in the classroom is closely connected to teacher's beliefs about the nature of mathematics and their pedagogy of teaching. This is particularly relevant to ICT and mathematics, where ICT activities are primarily dependent on individual teachers' initiatives (Watson, 1993). It also partially explains Watson's observation that individual teacher's positive ICT initiatives often do not reflect whole school mathematics teaching practices (this could be paraphrased as 'teachers who are technology enthusiasts are eccentrics').

We now turn our attention to computer algebra studies. Writing in 1995 Mayes (1997, 178) notes that the majority of dissertations and research studies published in refereed journals on the use of CAS in the learning and teaching of mathematics focus on student learning. There appeared to be no research focused on teachers using CAS. Studies not available to Mayes at the time have gone a little way towards redressing this imbalance. Zehavi (1996) focuses on in-service training where teachers work at problems at their own level, become aware of how technology can assist in their own mathematical thinking and then develop related materials at the student level. This model is partially incorporated into in-service training reported on by Lachambre & Abboud-Blanchard (1996). These authors also distinguish between four aspects of training: technical, scientific, cultural and professional. Heid (1995) and Zbiek (1995) show how computer algebra use can threaten teachers' perceived command of their subject knowledge and how teachers may bypass modes of effective teaching to ensure they exhibit a command of their subject knowledge to students in ICT lessons. Kendal and Stacey (1999) focus on both teachers and students and examine how teachers' "privileging", teaching styles and attitudes, differentially affected their students' learning in CAS lessons.

3. THE REGULAR USE OF ICT PROJECT

Stephen[2] and Steve were teacher-researcher members and John was the project co-ordinator of an Economic and Social Research Council funded project *Moving from Occasional to Regular Use of Technology in Secondary Mathematics Classes*. The project involved 13 teachers who made a commitment to move to regular use of technology in the 1998/99 school year. Most had some experience using technology in their classes but none had made extensive use of technology before. The project aimed to explore: patterns of teaching and learning; teachers' preparation and use of resources; teachers and students' attitudes and teachers' confidence ~ all over the course of one year.

The project's starting point was that many teachers in the UK finally had the opportunity to explore sustained use of technology with their classes (problems of accessing computers has limited use in the past). A very common recent past scenario that still applies in many schools is that, say, all the 14 year old in a school have a two week block of using, say, spreadsheets in mathematics lessons. Lessons usually start with keyboard/mouse skills, highlighting/ copy/ paste, file handling and basic Excel commands, e.g. fill down. By the end of the two weeks the students are just starting to use Excel for mathematical purposes but their block of time has finished and it is time for another class. The students' experiences in such cases are usually of *something we did* rather than learning to use a tool that can be used in further mathematical study. The project wanted to examine what happened when classes went beyond this minimal activity.

As the use of ICT is such a large area the project focused down on something common and manageable – using technology tools: spreadsheets, graphic packages and calculators and algebra and geometry systems. In an attempt to keep the project work as realistic as possible individual team members chose the tools they thought most appropriate for use with their classes. Stephen and Steve were the only teachers who chose to use a CAS.

The focus was really on the teacher, but learning is obviously crucial. A hope, rather than an aim, of the project was that students would reach a threshold of technology use that would produce instances where students who had been using, say, *Derive* and were working on a mathematics problem without *Derive* would say "Aha, *Derive* will help to explore this" and go off to a computer to explore. Such a threshold is suggested by the ImpacT report "...there may well be some minimum threshold of access, both frequency and over time, and type of classroom activity for such a contribution [to achievement] to become apparent ..." (Watson, 1993, 10).

The project may be viewed as adopting a 'naturalistic' approach (Lincoln & Guba, 1985). Apart from such unnatural activities as video-taping lessons every attempt was made to ensure that decisions came from teacher-researchers rather than university-based ones, that the situations were not manipulated nor the outcomes presumed. The data collected and analysed may be seen as a number of broad indicators, none of which providing evidence on their own. If data suggested a general trend across teachers and schools, then this would be noted but there was no prior expectation that practices could be generalized out of the school and class they existed in at a particular time.

Data was collected by teacher-researchers and university-based researchers. The project set out from the beginning that the two categories of researchers are likely to have different goals and interpretations and that neither has priority over the other. It is interesting to note that teachers appropriated their own research aims which were quite distinct from the formal aims of the project. Data collected included:

- observation of and accounts by teachers of their approaches to and use of technology
- weekly journal entry by teachers with relevant lesson plans and course materials
- teacher interviews
- classroom observation by teacher and university-based researchers
- student questionnaires and interviews
- use of student tasks as indicators of learning
- records of student performance in school-based tests

4. STEPHEN, STEVE, THEIR SCHOOLS AND THEIR CLASSES

Stephen and Steve both used *Derive* with 16/17 year old Advanced level (A-level[3]) classes. Stephen and Steve both teach in state schools in northern England. Stephen's school is a selective school for 11-18 year old students. Steve's school was (he has moved schools) a 'comprehensive' school for 13-18 year old students. Both Stephen and Steve have first class degrees in mathematics and they were in their second year of teaching when the project began (they were younger members of the project team, the mean number of years of service was 8). Both had spent a couple of hours playing with *Derive* on their own before the project began.

Stephen's class had 9 students and Steve's had 15 students. These are fairly typical A-level class sizes. Stephen had a suite of PCs in his teaching room. Steve had to take his class to a PC computer suite on the other side of the school. Stephen's class studied pure mathematics (the focus for the *Derive* work) with him from 7/9/98 - 30/3/99. Computer-based work made up 39 out of 112 lessons (35%) during this period. These were 'blocked': fairly regular until Christmas, then virtually no computer work until the end of February and then almost every lesson computer-based until the end of March. Like Stephen Steve focused the *Derive* work on the pure mathematics element of the course. Lessons at his school were twice as long as those Stephen's and his class spent 17 of their 51 'pure' lessons in the computer room. 33% of lessons may be seen as regular but, like Stephen's, they were blocked rather than regular. The lessons ran from 7/9/98 - 17/6/99 and 14 of the 17 *Derive*-based lessons were in the period from 21/11/98 - 11/2/99.

The project has had three 'specials' in the UK teachers' journal *Micromath* (Vol. 14/3 (1998), 15/2 and 15/3 (1999)). These included reports from the teacher-researchers at the beginning, at the midpoint and at the end of the project. Stephen and Steve's midpoint and end of project reports can be found in volumes 15/2 and 15/3.

5. STEVE'S EXPERIENCE

My intention was to use *Derive* as a teaching aid to cover the 'Pure 1' (P1) *Edexcel* (http://www.edexcel.org.uk/) syllabus, to reduce exposition time and to relieve students from 'tedious' calculations. My reasons for these choices are quite straightforward, though rather subjective. I was teaching the P1 module and *Derive* seemed appropriate for the content. Getting through the syllabus content traditionally involves considerable teacher exposition and I was keen to find ways to reduce this proportion of time devoted to exposition. There is a myth (I don't know how true it really is) that P1 is largely taught as a set of algorithms. I thought a stronger focus on concepts might arise from *Derive* work.

The areas for which I envisaged using *Derive* for are written in italics:

- Algebraic processing skills (polynomials and surds)
- Equations and inequalities (linear, simultaneous – both linear/ one quadratic, linear and quadratic inequalities)
- *Functions (notation, composite, inverse, modulus function, odd/even functions, transformation of functions)*
- Coordinate geometry (distance between points, gradient, equation of a straight line)
- Indices (rules of indices, fractional, negative, zero, *exponential function, natural logarithm function)*
- Sequences and series (sequences, series - including Σnotation, arithmetic and geometric series)
- Trigonometry (radian measure, basic trig, *graphs*, sin/cos/tan of any angle, *transformations of trig functions*)
- *Differentiation (gradient function, differentiation of* x^n *, exponential and logarithmic differentiation, increasing/decreasing functions, turning points, using differentiation to solve practical problems*
- *Integration (inverse of differentiation, boundary conditions, definite integrals, finding areas using integration)*

Numerical methods (absolute and relative errors, calculating with approximations, finding the roots of $f(x)=0$)

I expected that I would spend the majority of my time in the computer room. My primary resource was a textbook written specifically for the syllabus (which leads to the exam). The normal format in non-computer lessons was to explain topics myself and use the textbook for exercises. Every student in the class is asked to buy their own copy of the book, since homeworks are generally set from the exercises. For the non-*Derive* work, I invariably used the text book. For the *Derive* work, I chose not to use the text book (although as time went on I did occasionally use it for examples). The main reason for not using the textbook for *Derive-based work* is that I wanted to use *Derive* as a 'teach yourself' tool. In other words, I wasn't going to stand at the blackboard and teach a topic – I wanted to write a set of worksheets that would hopefully do the job for me.

My early work with *Derive* with this class was far from positive. After an couple of lessons introducing *Derive* we worked on functions: curve sketching and transformations, e.g. given the graph of $y=f(x)$, sketch the graph of $y=f(x\text{-}2)+5$. I spent a lot of time on early lesson plans and writing worksheets (1 to 2 hours per lesson instead of 5 minutes for non-computer lessons). We all found input and output notation difficult at times, e.g. how to differentiate a constant and $[x = y, x = - y]$ as the inverse of $y = x^2$. I found moving from the above list of topics I could do with *Derive*, to realising these with my class, difficult. One reason for this was simply the extra time it took ˜ in the early weeks of the *Derive*-based work we spent nearly every lesson in the computer room and the students were getting anxious and bored. I had to limit computer use to those times when I felt it would be really beneficial. Reflecting on the the reports of Heid (1995) and Zbieck (1995), noted above, the stress for me did not come from mathematical or technical concerns but from sensing that I was losing the interest and enthusiasm of the students.

My early worksheets were technology-focused, e.g. "Click and hold the left mouse button ..", and techno-maths focused, e.g. how express $\sqrt{}(x^2\text{-}1)$.Worksheets, however, quickly became straight-maths-focused and appeared to conform with my 'teach yourself' intent. I will illustrate the kind of work I set with a set of three worksheets designed to help introduce the idea of differentiation from first principles. These came from the 'happier period' when computer work was limited.

Worksheet one – basically an exercise using a spreadsheet: Define a function, say x^2. Then examine what happens to the gradient of the line joining point P to Q, as point Q is allowed to 'move down the curve' towards P.

The spreadsheet did all the hard work – evaluating the function at various points as the distance got smaller and smaller. By using the spreadsheet, the students were able to get a lot of results out very quickly – for, say, x=1,2,3,4,5. In doing this, patterns were quickly spotted. It was then a simple case of changing the formulae in a couple of cells to look at different functions.

x	h	x+h	f(x)	f(x+h)	gradient
1	0.5	1.5	1	2.25	2.5
1	0.3	1.3	1	1.69	2.3
1	0.1	1.1	1	1.21	2.1
1	0.05	1.05	1	1.1025	2.05
1	0.01	1.01	1	1.0201	2.01
1	0.001	1.001	1	1.002001	2.001
1	0.0001	1.0001	1	1.0002	2.0001

Worksheet two – this took the ideas developed using the spreadsheet and gave the students some notes on what was actually happening. This was now the first time that the concept of differentiation was introduced. The worksheet talked about some terminology and the notation used to represent derivatives and related this to *Derive* commands. The concepts, of course, were in the textbook but the links to *Derive* were not. I thus felt I had to write a worksheet with everything I wanted on it.

Worksheet three – Now that the students had encountered $\frac{dy}{dx}$, my intention was that they should use *Derive*'s algebraic processing power to examine $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ for various functions. At this stage I had not told them how to differentiate – they were going to find this out for themselves. My aim was for the hard slog to be taken out of the limit finding process. In the normal classroom environment, I would probably just look at x^2 in detail, going through each step on the board, and then leave the topic and just tell the class the general rule.

These lessons may be said to have worked well, but I still have reservations about them. As far as the functions topic was concerned, *Derive* was great - it even did the algebra for you. This brings up the issue of 'shouldn't the students be doing the algebra themselves, since they can't rely on *Derive* in an exam?' I suppose this isn't really a problem because the focus of the lessons was function transformations and it could be argued that the algebra could get in the way. *Derive* bypassed the 'hard slog' to enable a clearer vision of the results. With the calculus activities I encouraged them to write their results down in an orderly fashion but some, the lower attaining students, did not do this. So when they came to try to explain what they had found, they had no evidence to back up what they were trying to say. I need, too, to question the time taken ˜ two double lessons to arrive at results which could be obtained in half this time without *Derive*. I need to question the time taken for the early function work too: was it worth the 2-3 weeks learning the initial syntax just to see nice graphs appear on the screen? Packages such as *Omnigraph* could have been used just as effectively but with only a fraction of the time.

I ended the year questioning whether it was all worthwhile. I will probably use it again but as an occasional demonstration tool in the future.

Further comments on Steve's work

Steve's students filled in attitude questionnaires based on the French *Derive* attitude questionnaires detailed in Lagrange (1996). Lagrange constructed four attitudes towards *Derive* use: a worthless activity, a mine of information, a tool for learning and a tool to control calculations. Steve's class' responses clearly put their opinions in the last attitude category, a tool to ease calculations. Interviews with students support this view.

The class was video-taped on four occasions, before computer use and then at the beginning, the middle and the end of the computer-use period. Analysis of these tapes, using an amended version of the classroom analysis system of Beeby et al. (1979), showed significantly less teacher exposition, more student-centred work, less teacher-centred initiation of activities, less 'coaching' and more teacher-student 1-1(2) math talk in computer lessons. These indicators are consistent with Steve's aims to "reduce the amount of exposition which occurs and replace it with 'teach yourself' style worksheets".

6. STEPHEN'S EXPERIENCE

I wanted to use 'state of the art software' to follow the MEI 'Pure 1' mathematics A-level module (http://www.ocr.org.uk/develop/maths_b/mathsb.htm). I knew of, but was far from familiar with, *Derive* and it seemed an appropriate choice. I expected that I would also use graphic calculators, *Omnigraph* and Excel to support *Derive*-based work. I wanted my students to explore mathematics independently and I also wanted to increase their motivation to do mathematics.

The project used the term 'regular use of ICT'. I'm not sure that is a useful expression. There were times during the project, for example when my project class were studying mechanics, when ICT use seemed inappropriate and we didn't use it at all. There were other times, which I report on below, where we made really extensive use of ICT.

Instead of giving a step-by-step account of my work over the year I report on three matters: a double lesson on integration; an extended period where my students did coursework; my use, and non-use, of materials.

A lesson on integration

Half way through the year I was teaching integration involving natural logarithms and exponential functions to my class. I wanted to use *Derive*. Firstly I had to write my own worksheet (more on this in the third section). My aim was simple: instead of telling them the rules I gave them a list of integrals to evaluate on the computer which would hopefully lead them to work the rules out for themselves. Here is a sketch of the worksheet (I have not put every integral down):

> Evaluate the following integrals. Look carefully at the rule for the general case.
>
> $\int \frac{1}{x}\,dx \quad \int \frac{a}{x}\,dx \quad \int \frac{x}{x^2+1}\,dx \quad \int \frac{3x^2+2}{x^3+2x+5}\,dx$ Write down the general rule.
>
> $\int e^x\,dx \quad \int xe^{x^2}\,dx \quad \int (x+4)e^{x^2+8x}\,dx \quad \int (x^3+2x)e^{x^4+4x^2}\,dx$
>
> Write down the general rule.

The students immediately logged on and started the task. They were familiar with *Derive* by this stage and quickly spotted some of the easier patterns. Since their work was displayed on the screen line by line it made it easy for me to monitor their progress and offer suggestions or ask questions. When the patterns became harder to spot the students attitudes towards the work began to change. Some just 'turned off' and lost interest whereas others grouped together to compare answers and discuss what they were doing using very mathematical language. They used the computer to test and amend their ideas, which led one particular group to get very close to $\int \frac{af'(x)}{f(x)}dx = a\ln\{f(x)\}$.

This is fairly typical of many of my ICT lessons so far. Computers can motivate students and help to generate discussions amongst themselves. At the same time other students have an inherent dislike of computers and can become quickly bored. Computers can also introduce other problems. For example outputting answers the students did not yet understand for some of their own integrals. However this class are now used to this and they realised that their own input needed changing. Overall I felt that the benefits gained had made using *Derive* worthwhile.

Coursework

Shortly after the integration lesson I had to teach the class the techniques required for the 'Pure 2 coursework'. This coursework is an investigation into using numerical methods to solve equations. Students need to be able to use a change of sign method, fixed-point iteration and Newton-Raphson iteration. My involvement in the project led me to approach these topics through a variety of different software packages. The class were already familiar with Omnigraph and *Derive* but I also intended to use spreadsheets and graphical calculators.

My approach to teaching each of the three numerical methods was similar. I would start with the theory and then show the students how to reach solutions 'by hand', using only a calculator. This enables them to get a feel for how the methods converge, gets them used to the repetitive nature of the methods and helps them realise why computers will be useful. I would then introduce the software packages ˜ they could then select the software they thought was most appropriate. It is worth noting that this approach was very time consuming, both in terms of lesson time needed to do it properly and for me personally. Learning how to use the software meant many nights after school sitting at a computer figuring out the best way to do things and producing exemplar solutions to textbook questions. I encountered several problems but received support, in the form of technical assistance from the project group. I would probably have continued without this help but I think 'on-line' help from experienced colleagues is important when you are dealing with complicated software like *Derive* for the first time.

This time and effort paid off. Some of the students' coursework was of a very high standard. I found the spreadsheet to be very useful for this topic. It can be used easily for all methods and the screen can be easily edited to give concise printouts showing all relevant information. Many students with a PC at home were able to do work out of school. I found *Derive* to be rather cumbersome to use in comparison to the spreadsheet, e.g. it often requires quite technical knowledge to generate and format results and some quite complex commands (the use of *Iterates* in figure 1 below is an example). Omnigraph was invaluable for looking at graphs of functions, especially when the students were choosing their own equations.

This block of teaching and the subsequent coursework lasted several weeks. During this time the students were using computers almost every lesson. The students often helped each other, indeed problems encountered often generated mathematical discussions as students attempted to seek a solution. Of the nine students in the class, six chose to do their coursework extensively on computers. Out of the other three students, interestingly, two scored the lowest marks. The other student had hand written the whole coursework, using a graphic calculator to generate all answers and graphs.

Although I personally felt *Derive* to be cumbersome, the students who submitted the two best pieces of coursework, had predominantly used *Derive*. They both used *Derive* to produce some elegant solutions to the problems. They had interspersed their mathematics with explanatory text to good effect (see Figure 1).

#1: "Solving x^3/3 − x + 2=0 using Newton-Raphson iteration"

#2: PrecisionDigits := 21

#3: Branch := Real

#4: $F(x) := \frac{x^3}{3} - x + 2$

#5: $\frac{d}{dx}\left(F(x) := \frac{x^3}{3} - x + 2\right)$

#6: $x^2 - 1$

#7: $\text{ITERATES}\left(\left[n + 1, x - \frac{F(x)}{x^2 - 1}\right], [n, x], [0, -5], 6\right)$

#8:

$$\begin{bmatrix} 0 & -5 \\ 1 & -3.55555555555555555555 \\ 2 & -2.74576803739051883272 \\ 3 & -2.4162851095554892881 \\ 4 & -2.35714302586747185354 \\ 5 & -2.35530315181695447815 \\ 6 & -2.35530139760971373655 \end{bmatrix}$$

#9: "check for change of sign to confirm root"

#10: F(−2.355295)

#11: $2.90926725675416666743 \cdot 10^{-5}$

#12: F(−2.355305)

#13: $- 1.63817083325416666619 \cdot 10^{-5}$

#14: "Therefore x=−2.35530±0.000005"

Figure 1

It was interesting to compare two students' methods for the fixed point iteration section.

Student 1 scored 100%. He used *Derive* to generate all the iterations for his chosen equation $f(x) = e^x + x^2 - 3$. He rearranged the equation himself but used *Derive* to differentiate the rearranged function in order to check the gradient was between -1 and 1. He also established error bounds using *Derive*. To highlight key points he produced graphs, which he did on Omnigraph. He found this easier and quicker to use than the graphs on *Derive*.

Student 2 scored 88%. She used a spreadsheet to generate all the iterations for her chosen equation $f(x) = e^x - 2x^3 + 20$. She used *Derive* to draw her graphs,

Written materials

My primary resource for this course were textbooks designed for the modules but, with computer-based work, I wrote my own worksheets. I found writing worksheets the most time consuming aspect of being involved in the project. Knowing my frustration John lent me two very different books (Berry et al., 1993; Etchells et al., 1996). The first could be described as a traditional exposition of mathematics and the second as investigational. Although I used amended ideas from both books I did not find either particularly useful because they didn't fit in with the way I wanted to approach topics. I will try and explain my problems with them.

Berry *et al.* (1993) is not really a teacher's book as such - there are no lesson ideas, worksheets or anything directly useful. I found it useful though to work through it myself, discovering further capabilities of *Derive*, which I could then turn into ideas for lesssons. One particular example of this is using *Derive* to perform iterations. The ITERATE command is reasonably straightforward for the pupils to use once it had been explained to them, however it doesn't present answers in a quickly readable form. It is better to set up the iteration as a vector, so that each step of the iteration is numbered. This obviously makes the initial command more complex. The pupils had to copy it 'parrot fashion' and I had to highlight which numbers did what. Two students did then use this command with good effect in their coursework, although during the initial lessons other students were struggling to comprehend this command fully.

Etchells *et al.* (1996) is written specifically for teachers, full of lesson ideas. There are plenty of photocopiable worksheets, teacher notes and help sheets. Despite all this, I did not find it immediately useful. One reason is that I am teaching a modular course the calculus topics have been split into separate modules. There is a (necessary) tendency to teach for the exam. So, for example, the basic idea of differentiation occurs in the P1 module whereas the second derivative doesn't show up until P2. Several of the worksheets in the book contained ideas which overlapped modules, and so were not appropriate until the later modules were being taught. Other activities in the book, which although they looked good, were not directly related to the topics being covered. One example of this is the activity 'Multiplying Straight Lines'. I believe this to be a worthwhile activity, but it is definitely an 'extra'and several of the ideas being beyond the immediate syllabus. (I have since used this as an 'end of term' activity but I found it easier with Omnigraph because I didn't need to keep switching from algebra to graph windows.)

Further comments on Stephen's work

Stephen reports that his students responded well to *Derive*. Their responses to the Lagrange (1996) questionnaire, however, clearly places them in Lagrange's 'tool to ease calculations' attitude category. Analysis of video-tapes of lessons shows remarkably little change in the Beeby et al. (1979) categories, from non-computer lessons, of classroom behaviours in computer lessons.

7. DISCUSSION

For the purpose of communication we group our reflections on our experiences into four categories: time, materials, pressures and *Derive*.

Time

Incorporating *Derive* into lessons involved considerable extra work/time: becoming basically competent with it; going through the syllabus and finding suitable topics; planning lessons in much greater detail than would normally be the case; writing and testing worksheets. This extra work was definitely biased towards the beginning of the course but it remained throughout the year. We may have been extra conscientious because we were involved in a project but we believe the situation would have been more or less similar if we had not been. There is an argument that any new teaching resource involves extra effort on the teacher's part. We nevertheless believe that *Derive*, used as a regular resource, is at the upper end of the 'effort' scale for mathematics software. We return to this in the section on *Derive* below. Time issues for teachers using technology may have a simple explanation but they are important to note because the impression is often given that technology is something that it can be readily incorporated into classroom work.

Another time issue, but not one related to extra work, is time to get 'a feel' for how to use *Derive*. Going through the syllabus and finding suitable topics for *Derive* use is one thing, having a sense of how they might 'work' is another. Take Steve's original list with areas envisaged for *Derive* use. Much of this work was not done. There is an argument, again, that this happens with any new development but, again, we believe that *Derive* is at the upper end of the scale for mathematics software. Perhaps this is partially due to the enormous potential of *Derive* for this kind of mathematics. It is not just something, like a graph plotter, that does a specific task, it can do just about everything. This may be a selling point for *Derive* but, for the teacher, it can present real problems.

Materials

Steve commented that his early worksheets were technology-focused, e.g. "Click and hold the left mouse button ..", and techno-maths focused, e.g. how to express $\sqrt{}(x^2-1)$, but that they then became mathematics-focused. We think this is a common pattern for teachers when they begin to use technology ˜ almost any technology. There is a sense, too, in which this is natural: if you are going to use technology to support the learning of mathematics, then you first need to learn how to use the technology.

But once this hurdled is over, why continue using worksheets ˜ why not use the textbook? Steve claims that this resulted from a desire to use *Derive* as a 'teach yourself' tool. In Stephen's case he simply found that the textbook and *Derive* did not 'fit'. It is difficult to isolate reasons for this. There are a number of possibilities. It may be related to 'time to get a feel for how to use *Derive*' noted above. It may arise from a desire to 'lead' students' use of *Derive* because it offers so much scope and the teacher wishes the students to go down a particular route they believe is beneficial to learning. It may also be because written mathematics and *Derive*-based mathematics are two different forms of mathematics.

Whatever the reason, why did Stephen find the two textbooks of little use? They both directly addressed using computer algebra. They were both aimed at this level of mathematics. They were offered to Stephen as different types of resources ˜ one quite traditional, one investigational. Stephen stated that the first had no lesson ideas and the second presented activities that crossed modular syllabuses. Reflecting on this now we believe there is something more that relates to the power of *Derive*, to leading students routes and to getting a feel for *Derive*-based mathematics. It is no more than an hypothesis but we think that teachers who plan to incorporate significant use of computer algebra in their teaching are presented with a re-evaluation of the mathematics they were taught and are familiar with. These re-evaluations are quite specific to the individual and someone elses 'route' is not easy to accomodate.

Pressures

We start by noting that formal assessment in England permeates every aspect of teaching at every level of schooling. When Stephen states "there is a (necessary) tendency to teach for the exam" he speaks for virtually every teacher in England. We are not overstating this case. This situation has arisen in the last 10 years. The main pressure on A-level mathematics teachers using computer algebra is the pressure to ensure that it supports formally assessed work.

One aspect of the time concerns noted above is the pressure that lesson time needs to be directed towards assessable outcomes. We all felt this. Indeed the project had a moral clause that ICT work should be set aside if it was likely to jeopardise students' scholastic interests, i.e. exams results. The students feel this too ˜ they have grown up in an assessment ethos. Steve reports that stress came from sensing that he was losing the interest and enthusiasm of the students. Part of this, he feels, arises from exam concerns. Indeed, most of the students agreed that *Derive* would not help them in the exams.

Hand in hand with this emphasis on assessment is a hierarchical structuring of the curriculum. One aspect of this hierachical structuring in A-level is modular courses. Stephen comments that the basic idea of differentiation occurs in the P1 module whereas the second derivative doesn't show up until P2. Although this is not a stress-inducing constraint it is a factor that affects attitudes to using *Derive*.

Hoyles (1992, 40) argues that "all beliefs are situated ˜ dialectical constructions, products of activity, context and culture". It is tempting to dismiss this use of language as 'psycho-jargon'. But surely the pressures arising from assessment and curriculum constraints, and the (reflected) reactions of students affected our attitudes to and beliefs about the usefulness of *Derive* in our classrooms.

Derive

Was *Derive* itself an important factor in shaping our experiences? Other tools used by project team members included graphic calculators, graphics packages, spreadsheets and Geometers' Sketchpad. Project work suggested that spreadsheets and graphic applications have an immediacy which *Derive* and Sketchpad lack. 'Immediacy' is simply a term we use here. It has several levels of meaning.

- Software is immediate if you can use it quickly. The graphic package *Omnigraph* was immediate, in this sense. All project members, teachers and students, were using it within five minutes. *Derive* is not immediate in this sense. Both classes had to have several lessons devoted to learning to use *Derive* and further command-based learning was a feature of later lessons.
- Software is immediate if you can proceed with a task without getting caught up in technicalities. *Derive* is not immediate in this sense. For example, in one of Steve's lessons on functions it became necessary to specify inverse functions. $f^{-1}(x)$ does not work. It is necessary to write $y = f(x)$ and rearrange to get x as a function of y. This caused the mathematical focus of the lesson to be put aside in order to focus on how to perform the technical operation.
- Software is immediate if its place in the mathematics being studied is clear. This level of immediacy is complex and intertwined with many person/situation-specific factors: the 'transparency' of the mathematics, the transparency of the software in dealing with this mathematics, the mathematical and software specific technical facility of the teacher and of the student and the size of the mathematical task. One of the problems with *Derive*, with respect to 'transparency' is, as discussed above, its enormous potential ˜ its power appears to work against transparent usage.

One problem may have been the centrality of *Derive* in Stephen and Steve's use of ICT (any implied criticism here should reflect on the project rather than Stephen and Steve). Although both used other ICT tools, *Derive* was the main ICT tool. There appears to be a dilemma for mathematics teachers about what tools to use. It can be argued that a number of tools, each with its own specific strength, should be encouraged. A possible disadvantage with this view is that students may end up confused and unable to use any one fluently. A focus on a single tool, however, may lead to its forced use when another tool can perform a task better. Steve's question 'was it worth the 2-3 weeks just to see nice graphs when *Omnigraph* could do it in a fraction of the time' is relevant here. There is also an argument as well that students should have an important part in deciding which tool to use. Stephen's comments on the tools his students chose for the fixed point iteration coursework task show that very different intelligent choices may be made.

We commented in section 3 that we hoped that students would reach a threshold of *Derive* use where they would, without being directed to, use *Derive* in the course of their mathematics. Apart from the couple of students in Stephen's group who used it in their coursework (where they were encouraged to use technology) there were no instances of this. In the case of Steve's class, however, they would have had to travel to the other side of the school to do so.

8. CONCLUSION

Stephen and Steve intended to use *Derive* on a regular basis but, for reasons of syllabus and of getting through the year's work, they used it in blocks. This has practical implications for booking computer rooms. It took them both some time to get a feel for where and how it could be useful. It took their students sometime before they used it with any degree of ease. Their students, moreover, responded with varying degrees of enthusiasm to this use of computers. Coursework aside it did not appear that their students crossed a threshold of *Derive* use that involved spontaneously using it to assist with their mathematics.

Using *Derive* involved a lot of additional work for both Stephen and Steve. This was front-end loaded and involved personal familiarization with *Derive*, planning lessons (in much greater detail than 'ordinary' lessons) and writing worksheets. Worksheets were used much more than in ordinary lessons because the standard textbooks for the courses were not seen as useful for *Derive*-based lessons. *Derive*-focused published material did not help this situation, at least in Stephen's case, because the material was not presented in a manner that he felt natural with. Technical and mathematical problems were encountered during the course of the year but, unlike experiences reported by Heid (1995) and Zbieck (1995), these did not cause the teachers any reported anxiety (perhaps due to their personal confidence in the own mathematical powers). Anxiety, however, was experienced, by Steve, when he felt the class was getting bored with computer-based work or that they were falling behind on the scheme of work.

The upshot of this for experienced technology enthusiasts is the need to be careful about putting forward solutions as exemplars for others. Making extensive use of technology appears to be a way to enable students to use technology as a tool for doing mathematics, as opposed to a *one*-off experience done for the sake of covering technology. But making regular use of *Derive* in mathematics teaching is a difficult and time consuming job. It may, moreover, be perceived as something that is neither rewarding nor desirable.

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[1] ICT: information and communications technology. This is the current UK term used for digital technology.
[2] Writing a three-person paper that examines the practices of two of the authors presents stylistic problems. We adopt a style of using first names, "we" and "I" as appropriate.
[3] Advanced level (A-level) Mathematics is the most common senior public examination for students in the UK. It covers considerable algebra and calculus of a single variable. There are five pass grades, A to E. Examinations are set and marked by independent institutions called Examination Boards.

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BIOGRAPHICAL NOTES

Stephen Lumb and Steve Mulligan both obtained first class degrees in Mathematics from the University of Leeds. They are both enjoying their third year of teaching mathematics to 11-18 year olds in schools close to Leeds. John Monaghan works at the Centre for Studies in Science and Mathematics Education, University of Leeds.